NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 4241

AN APPROXIMATE METHOD FOR DESIGN OR ANALYSIS OF TWO-DIMENSIONAL SUBSONIC-FLOW PASSAGES

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SUMMARY

A method has been developed for the design and analysis of two-dimensional subsonic-flow passages with isentropic nonviscous flow. The method is based on the relation between the pressure change across a stream tube and the centrifugal force resulting from the curvature of the flow. Precomputed charts can be drawn which eliminate subsequent calculation for a given upstream Mach number. The method is limited by the accuracy with which the radius of curvature of the streamlines can be determined.

In an example the method was applied to the design of an expanding elbow at each of two Mach numbers. Two elbow contours and their surface pressure distributions were obtained.

INTRODUCTION

Much experimental work on internal-flow systems has consisted of tests of more or less arbitrarily laid out duct components to determine regions of separated flow, pressure losses, and flow distortion at the exit. This research could be conducted in a more effective manner if a method were available to design some of these duct components to avoid regions of local speedup and steep adverse pressure gradients. Furthermore, if the flow in existing components could be analyzed, the analysis would furnish a basis for comparison of ideal and actual performance and could be used as a basis of elimination of unpromising components from an experimental program.

Several methods for analyzing the flow in subsonic duct systems are available. Flügel's stream-filament method discussed in reference 1 was utilized in references 2 and 3 by making arbitrary assumptions about the variation of the radius of curvature. The method was used in reference 4 for the analysis of the flow in three-dimensional passages and in reference 5 for the design of three-dimensional passages. A wire-mesh plotting device for incompressible flow is described in reference 6, and in reference 7 a device consisting of cams combined with a wire mesh is described which is applicable to compressible flow.

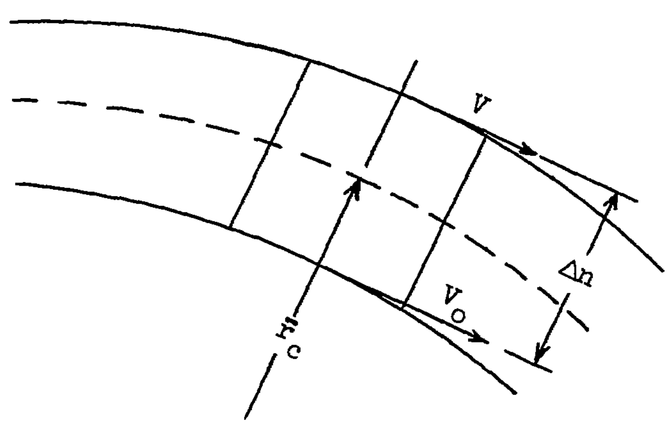
In the present report a method is developed for the design and analysis of two-dimensional subsonic-flow passages with isentropic nonviscous flow. The method parallels the three-dimensional treatment of references 4 and 5. Restriction of the treatment to two-dimensional-flow problems permits the use of precomputed charts from which the values required can be read directly. The charts can be used for design or analysis of flow passages with the assumption of isentropic, subsonic, nonviscous flow.

DEVELOPMENT OF METHOD

The method developed in the present report is based on a relation between the pressure difference across a small stream tube and the centrifugal force per unit area caused by the flow curvature:

$$dp = -\rho \, dn \, \frac{V^2}{r_c}$$

In the following diagram is shown a small stream tube of width Δn and average radius of curvature $\bar{r}_c$ (all symbols are defined in appendix A):



The ratio of the velocities at the intersection of a normal to the boundaries of the stream tube is derived in appendix B and given by equation (1):

$$\frac{V}{V_o} = e^{\Delta n/\bar{r}_c} \tag{1}$$

Since no density values are involved, equation (1) is applicable to either incompressible flow or compressible flow.

Continuity for two-dimensional flow requires that between any station and an upstream station where the flow is rectilinear

$$\overline{\rho}\overline{V}\Delta n = \rho_u V_u \Delta n_u \tag{2a}$$

for compressible flow and

$$\overline{V}\Delta n = V_u \Delta n_u \tag{2b}$$

for incompressible flow.

If $\overline{V}$ is taken as $\overline{V} = \frac{1}{2}(V_o + V)$, then

$$\frac{\overline{V}}{V_u} = \frac{\frac{V_o}{V_u} + \frac{V}{V_u}}{2} \tag{3}$$

Velocity and density for isentropic compressible flow are related by equation (63) of reference 8:

$$\frac{\overline{\rho}}{\rho_t} = \left[1 - \frac{\gamma - 1}{2}\left(\frac{\overline{V}}{a_t}\right)^2\right]^{\frac{1}{\gamma-1}} \tag{4}$$

By use of elementary isentropic relations and equations (2a), (3), and (4) it can be shown that

$$\left(\frac{\Delta n}{\Delta n_u}\right)^{-1} = \frac{\frac{V_o}{V_u} + \frac{V}{V_u}}{2}\left\{1 + \frac{\gamma - 1}{2} M_u^2 \left[1 - \left(\frac{\frac{V_o}{V_u} + \frac{V}{V_u}}{2}\right)^2\right]\right\}^{\frac{1}{\gamma-1}} \tag{5}$$

For consistency equation (1) is rewritten as

$$\frac{V}{V_u} = \frac{V_o}{V_u} e^{\frac{\Delta n}{\Delta n_u} \frac{\Delta n_u}{\overline{r}_c}} \tag{6}$$

Equations (5) and (6) are obviously not easily solved for V/V_u and $\Delta n/\Delta n_u$ for given values of V_o/V_u, $\Delta n_u/\bar{r}_c$, and M_u. A chart can be made, however, to represent equations (5) and (6) for any given upstream Mach number by assigning a value to V_o/V_u and using several values of $\frac{\Delta n_u}{\bar{r}_c}\frac{\Delta n}{\Delta n_u}$ to obtain values of V/V_u, $\Delta n/\Delta n_u$, and $\frac{\Delta n_u}{\bar{r}_c} = \frac{\Delta n_u}{\bar{r}_c}\frac{\Delta n}{\Delta n_u}\left(\frac{\Delta n}{\Delta n_u}\right)^{-1}$. Thus lines for constant values of V_o/V_u can be drawn on a chart with $\Delta n/\Delta n_u$ for ordinate and $\Delta n_u/\bar{r}_c$ for abscissa. By cross-plotting, lines of constant values of V/V_u can be superposed on this chart. However, a simpler method is described in appendix C whereby a chart can be easily constructed for a given upstream Mach number. From this chart values of $\Delta n/\Delta n_u$ and V/V_u can be read directly for given values of V_o/V_u and $\Delta n_u/\bar{r}_c$. Figure 1 shows charts for incompressible flow. Figures 2 and 3 show charts for compressible flow for upstream Mach numbers of 0.40 and 0.85, respectively. If a value of Δn_u can be decided upon for a particular case, the corresponding curves can be relabeled with values of Δn and $1/\bar{r}_c$. In figure 4, which is for an upstream value of Δn of 2.00 inches, the values of Δn are given in inches and values of $1/\bar{r}_c$ in (inches)$^{-1}$. The lines of this figure are the same as those of figure 3 but have different labels.

The accuracy of this method depends in part on the accuracy with which $1/\bar{r}_c$ can be determined and decreases as $1/\bar{r}_c$ becomes very large or very small. Discontinuities in r_c for the starting boundary should be kept to a minimum magnitude. The correct sign for $1/\bar{r}_c$ must be used. The sign of $1/\bar{r}_c$ is the same as that of the second derivative of the curve and can usually be determined by inspection. If the curve is concave when viewed from the direction from the boundary to streamline 1, the reciprocal of the radius of curvature is negative. Inadvertent use of the wrong sign will immediately be apparent because the direction of pressure change indicated by the change in velocity across a stream tube will not be consistent with the direction of turning. It is considered that the accuracy decreases for successive streamlines because the measured values of $1/\bar{r}_c$ depend more on the artistry of fairing in the streamlines through the $\Delta n/\Delta n_u$ points.

APPLICATION OF METHOD

Design of Duct Shape

Use of the charts for design purposes is essentially as follows: A boundary or other streamline along which a chosen velocity variation is prescribed is laid out and perpendiculars are erected at a suitable number of stations. If the boundary has been determined analytically, it may be possible to compute the radius of curvature at each station and estimate the average radius of curvature for the stream tube. Otherwise, some other means, such as the radometer shown in figure 5 and described in appendix D, is required to determine the radius of curvature. For each station, values of $\Delta n/\Delta n_u$ and V/V_u are read at the proper values of V_o/V_u and $\Delta n_u/\bar{r}_c$. The first streamline can now be drawn in and the average radius of curvature determined by measurements. The new values of $\Delta n_u/\bar{r}_c$ based on the measured value of $1/\bar{r}_c$ must be used to determine new values of $\Delta n/\Delta n_u$ and V/V_u. This iteration procedure must continue until the values of $\Delta n/\Delta n_u$ are essentially unchanged by successive values of $\Delta n_u/\bar{r}_c$ obtained from measurements. The number of repetitions required is reduced by the fact that changes in the values of $\Delta n_u/\bar{r}_c$ do not give large changes in the values of $\Delta n/\Delta n_u$. For example, a 50-percent increase in $\Delta n_u/\bar{r}_c$ (from 0.030 to 0.045) in figure 3(c) at V_o/V_u of 0.99 gives a change in $\Delta n/\Delta n_u$ of only 0.5 percent.

The final values of V/V_u are now used as values of V_o/V_u for the next streamline. In constructing the next streamline the perpendicular along which $\Delta n/\Delta n_u$ will be measured is laid off from the midpoint of the perpendicular used in the previous stream tube. The values of V/V_u for the final streamline can by recourse to tables for compressible flow be converted to obtain pressure-distribution curves.

Analysis of Flow Quantities

The method of analysis is used when the inner and outer boundaries are prescribed. The procedure to be described is given as one possibility, but may, of course, be altered to suit the individual situation.

After stations are selected along one of the boundaries, an approximation to a line perpendicular to the streamlines must be drawn in for each station. Drawing in estimated streamlines might be of some assistance in making this approximation. Selection of the number of stream tubes to be used determines the upstream value of Δn. The total length of the line perpendicular to the streamlines is measured for each station.

At each station average values of $\Delta n_u/r_c$ must be determined for each stream tube either analytically or by measuring the values of $\Delta n_u/r_c$ at both boundaries and plotting a reasonable curve between the two values.

With the streamlines numbered from 0 for the starting boundary up to the other boundary, which will have a number equal to the number of stream tubes, a tabulation can be made for each station as shown in the following table:

Stream tube number	$\frac{\Delta n_u}{\bar{r}_c}$	$\frac{V_o}{V_u}$	$\frac{\Delta n}{\Delta n_u}$	$\frac{V_o}{V_u}$	$\frac{\Delta n}{\Delta n_u}$
1					
2					
3					
.					
.					
.					
Summation of $\frac{\Delta n}{\Delta n_u}$ values					

In the table the stream tubes are identified with the streamline whose location is being determined. The values of $\Delta n_u/\bar{r}_c$ already determined are written in for each stream tube.

A value of V_o/V_u is needed for the first boundary at each station. It can be obtained from the precomputed charts at the proper value of $\Delta n_u/\bar{r}_c$ and an estimated value of $\Delta n/\Delta n_u$. This value of V_o/V_u is written in the first column headed V_o/V_u for stream tube 1. The precomputed chart is used to determine values of $\Delta n/\Delta n_u$ and V/V_u for stream tube 1; however, the value of V/V_u for stream tube 1 is written in as the value of V_o/V_u for stream tube 2. This procedure is continued until there is a value of $\Delta n/\Delta n_u$ for each stream tube and one more velocity ratio than there are stream tubes. The sum of the computed values of $\Delta n/\Delta n_u$ is compared with the sum of the values of $\Delta n/\Delta n_u$ measured on the layout. If these sums are not equal, a new value of V_o/V_u for stream tube 1 is inserted in the second column headed V_o/V_u and new values are picked from the precomputed chart. The new initial value of V_o/V_u is obtained by multiplying the value previously assumed by the ratio of the sum of the computed values of $\Delta n/\Delta n_u$ to the sum of the measured values of $\Delta n/\Delta n_u$. This routine may have to be repeated if the sums of $\Delta n/\Delta n_u$ are still not sufficiently close.

The computed values of $\Delta n/\Delta n_u$ are now used to plot streamlines. For each station, perpendiculars to the streamlines are constructed in the manner described for the design method. Values of $\Delta n/\bar{r}_c$ determined from measurements are inserted in the table and a value of the summation of $\Delta n/\Delta n_u$ is determined from measured values of $\Delta n/\Delta n_u$. A value of V_o/V_u for the first boundary is obtained from the precomputed charts and values of $\Delta n/\Delta n_u$ and V/V_u are successively obtained for each stream tube. This procedure is repeated with a new value of V_o/V_u for the start if the sum of the computed values of $\Delta n/\Delta n_u$ does not equal the sum of the measured values. If necessary, new streamlines are drawn and the process repeated until the streamlines plotted from computed values of $\Delta n/\Delta n_u$ are essentially unchanged because of the use of measured values of $\Delta n_u/\bar{r}_c$. The required number of repetitions is reduced by the fact that, as has been pointed out, changes in $\Delta n_u/\bar{r}_c$ do not cause proportionate changes in $\Delta n/\Delta n_u$.

Examples

The design method was used to lay out an expanding 90° elbow (fig. 6) for an upstream Mach number of 0.85. The area ratio was 1.37, and the space used up was to be confined to moderate bulges outside that required by a nonexpanding elbow of mean radius three times the inlet width. It was thought that use of an entering Mach number higher than 0.85 might result in some supersonic flow in the elbow.

A modified sine curve was used for the inner boundary in order to have zero values of $1/r_c$ at the beginning and the end of the turn. It was assumed advantageous to spread the adverse pressure gradient evenly over the entire length of the inner surface.

For the layout an upstream width of 10 inches was divided into five stream tubes having equal flow.

It was found that some drop in pressure on the inner wall in the region of the start of the turn was required in order to avoid a necking-in of the outer wall. This necessity for a drop in pressure can be explained as follows: Consider first a station a short distance past the start of the turning of the inner surface. The assumed distributed pressure rise gives a small increase in pressure over the upstream value. The turning requires that the next streamline out have a slightly greater increase in pressure and that, consequently, the stream-tube width be increased over the upstream value. By this same reasoning each successive stream-tube width at this station will be slightly greater than that of the preceding one. These increases in stream-tube width move the outer boundary at this station outside of a line which is an extension of the outer straight inlet boundary. This outward movement of the boundary means that at some previous station the outer boundary had to curve outward to reach this point. Now consider this previous station. At the inner boundary there is no curvature and the pressure has not changed from the upstream value. For the next streamlines the pressure and stream-tube width are unchanged up to the region where the outer streamlines are curved outward; therefore, each of the last streamlines must have a lower pressure than the one before it and the stream-tube widths are successively decreasing. The result is that the outer boundary is pulled inside the line of the straight inlet. It was assumed that a decrease in area ahead of a diffusing bend was not favorable. The decrease in area was eliminated by applying an inner boundary pressure lower than the upstream pressure to both of the stations previously discussed. At the station beyond the start of the bend this lower pressure resulted in moving all the streamlines inward and reduced the amount that the outer boundary had been outside the inlet line. The outward curvature of the outer boundary at the preceding station was therefore reduced. The application of the lower inner boundary pressure and the resultant moving inward of the streamlines gave a curvature in the direction of the bend to the first streamlines of the station previous to the start of the inner-wall bend. The result at this station was that the stream tubes near the inner walls were thinner than the upstream values. This result permitted the stream tubes in the center to be wider and the pressures to be higher and consistent with curvature away from the center of the passage toward each boundary. The prescribed inner-wall pressure was modified several times and the design was restarted until the result was an outer boundary which did not reduce the cross-section width before the start of the bend.

The resultant elbow and streamlines are shown as the solid lines in figure 6. The variation of velocity, stream-tube width, and radius of curvature along the streamlines is indicated in figure 7. The corresponding pressure distributions over the inner and outer surfaces are shown as the solid lines in figure 8.

For comparison with this elbow designed for a Mach number of 0.85 the design of an elbow for an upstream Mach number of 0.40 was started. The Mach number of 0.40 was chosen as being in the region of effects midway between the example Mach number of 0.85 and incompressible flow. The same inner-wall shape and area ratio were used. It was found that a larger drop in pressure at the start had to be tolerated. This drop in pressure is due in part to the fact that a greater area change is needed at the lower Mach number for a corresponding change in pressure.

Toward the outlet of the elbow it became very difficult to modify the inner-wall velocity distribution in such a way as to get the outer boundary to come out smooth and fair with the outlet line and the first half of the outer boundary. The first half of the outer boundary was, therefore, faired into a curve arbitrarily taken from the last part of the outer boundary of the elbow designed for a Mach number of 0.85, and the analysis method was applied.

The use of the analysis method permitted the intermediate streamlines and the pressure distributions over the inner and outer surface to be obtained. The boundaries and streamlines are shown as the dashed lines in figure 6. The surface pressure distributions are shown as the dashed lines in figure 8.

In the examples a 10-inch inlet and a value of Δn_u of 2.00 inches were used. It was found convenient to use charts similar to figure 4, which was made for a value of Δn_u of 2.00 inches, and to use a radometer which gave direct readings of $1/r_c$ in $(\text{inches})^{-1}$. It was found helpful to make plots of $1/r_c$ and Δn against linear distance along the boundary for the design method. For the analytical method plots were made of $1/r_c$ and Δn against n.

A rough check was made on the effects of the number of stream tubes into which the passage is divided. Station 11 of the example for an upstream Mach number of 0.40 was investigated by the analytical method with 10 stream tubes instead of 5. The solid line in figure 9 gives the pressure variation across the passage with the use of 5 stream tubes. The dashed line in figure 9 gives the pressure variation for 10 stream tubes. The greatest difference was 0.010q at the inner boundary. Consistency would indicate an approximately equal change at the adjacent stations so that the effect on the pressure gradient would be nil. The largest shift in streamline location was 0.02 inch. In this case the extra work required to use 10 stream tubes instead of 5 would apparently not be justified.

CONCLUDING REMARKS

In the present report a method has been developed for the design and analysis of two-dimensional subsonic-flow passages with isentropic nonviscous flow. The method is based on the relation between the pressure change across a stream tube and the centrifugal force resulting from the curvature of the flow. Precomputed charts can be drawn which eliminate subsequent calculation for a given upstream Mach number. The method is limited by the accuracy with which the radius of curvature of the streamlines can be determined.

In an example the method was applied to the design of an expanding elbow at each of two Mach numbers. Two elbow contours and their surface pressure distributions were obtained.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
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APPENDIX A

SYMBOLS

a	speed of sound, ft/sec
g	acceleration due to gravity, ft/sec^2
M	Mach number
n	distance along normal to streamline
Δn	distance between adjacent streamlines along normal
p	static pressure, lb/sq ft
q	dynamic pressure, $\frac{\rho}{2} V^2$, lb/sq ft
r_c	radius of curvature
V	velocity, ft/sec
V_o	velocity at first streamline bounding a stream tube, ft/sec
ρ	mass density, lb-sec^2/ft^4
γ	ratio of specific heats

Subscripts:

t	stagnation conditions
u	conditions upstream where flow is not turning
0,1, . . .	points at succeeding streamlines along normal

A bar over a symbol denotes an average value representative of the stream tube.

APPENDIX B

DERIVATION OF VELOCITY EQUATION

The velocity equation used in the method of the present report is derived from a relation between the pressure difference across a small stream tube and the centrifugal force per unit area caused by the flow curvature:

$$dp = -\rho \, dn \, \frac{V^2}{r_c}$$

or

$$\frac{dp}{\rho} = -\frac{V^2}{r_c} \, dn \tag{B1}$$

From Euler's equation for steady one-dimensional flow,

$$\frac{dp}{\rho} = -V \, dV \tag{B2}$$

Combining equations (B1) and (B2) gives

$$\frac{dp}{\rho} = -V \, dV = -\frac{V^2}{r_c} \, dn$$

Dividing by V^2 yields

$$\frac{dV}{V} = \frac{1}{r_c} \, dn$$

$$\int_{V_o}^{V} \frac{dV}{V} = \int_0^n \frac{1}{r_c} \, dn$$

$$\log_e\left(\frac{V}{V_o}\right) = \int_0^n \frac{1}{r_c} \, dn$$

$$\frac{V}{V_o} = e^{\int_0^n \frac{1}{r_c} dn}$$

If Δn, the distance between the streamlines, is small, an average value of r_c may be used and,

$$V = V_o e^{\Delta n / \bar{r}_c} \tag{B3}$$

Equation (B3) could have been obtained from equation (3) of reference 5 by eliminating the last term of equation (3) which is zero in the case of two-dimensional flow.

APPENDIX C

CONSTRUCTION OF PRECOMPUTED CHART FOR SELECTED UPSTREAM MACH NUMBER

For a given upstream Mach number a chart can be constructed from which values of $\Delta n/\Delta n_u$ and V/V_u can be read directly for given values of V_o/V_u and $\Delta n_u/\bar{r}_c$. In a plot of V/V_u against V_o/V_u, a line through the origin with a slope of 1 represents the case of no curvature, $\frac{1}{\bar{r}_c} = 0$. For incompressible flow (fig. 1), equation (2b) can be used to spot values of $\Delta n/\Delta n_u$ on this line. For compressible flow at a given upstream Mach number (figs. 2 and 3) tables for one-dimensional flow (ref. 8) can be used to determine values of $\Delta n/\Delta n_u$. Since $\Delta n/\Delta n_u$ determines $\bar{V}/V_u$ for a fixed upstream Mach number, the averaging equation

$$\frac{\bar{V}}{V_u} = \frac{\frac{V_o}{V_u} + \frac{V}{V_u}}{2}$$

shows that lines drawn through the points on the line for $\frac{1}{\bar{r}_c} = 0$ with a slope of -1 each represent conditions for a constant value of $\Delta n/\Delta n_u$.

Points spotted on several of the lines of constant values of $\Delta n/\Delta n_u$ for a chosen value of $\Delta n_u/\bar{r}_c$ permit drawing in a line for the constant value of $\Delta n_u/\bar{r}_c$. These values are determined as follows: For a constant value of $\Delta n/\Delta n_u$ it can be shown that the average velocity $\bar{V}/V_u$ is equal to the no-curvature velocity $\left(\frac{V_o}{V_u}\right)_{\frac{1}{r_c}=0}$. Use of this fact in combination with equations (3) and (6) for a constant value of $\Delta n/\Delta n_u$ gives

$$\frac{V_o}{V_u} = \frac{\left(\frac{V_o}{V_u}\right)_{\frac{1}{\bar{r}_c}=0}}{\frac{1}{2}\left(1 + e^{\frac{\Delta n_u}{\bar{r}_c}\frac{\Delta n}{\Delta n_u}}\right)}$$

The change in V_o/V_u due to curvature when $\Delta n/\Delta n_u$ is held constant is then

$$\left(\frac{V_o}{V_u}\right)_{\frac{1}{\bar{r}_c}=0} - \left(\frac{V_o}{V_u}\right) = \left[1 - \frac{1}{\frac{1}{2}\left(e^{\frac{\Delta n_u}{\bar{r}_c}\frac{\Delta n}{\Delta n_u}} + 1\right)}\right]\left(\frac{V_o}{V_u}\right)_{\frac{1}{\bar{r}_c}=0}$$

$$\left(\frac{V_o}{V_u}\right)_{\frac{1}{\bar{r}_c}=0} - \left(\frac{V_o}{V_u}\right) = \frac{e^{\frac{\Delta n_u}{\bar{r}_c}\frac{\Delta n}{\Delta n_u}} - 1}{e^{\frac{\Delta n_u}{\bar{r}_c}\frac{\Delta n}{\Delta n_u}} + 1}\left(\frac{V_o}{V_u}\right)_{\frac{1}{\bar{r}_c}=0} \tag{C1}$$

Three or four values of Δn suffice to establish a line for a given value of $\Delta n_u/\bar{r}_c$ since it is nearly a straight line. The complication of using equation (C1) can be greatly reduced by use of the fact that the term $\dfrac{e^{\frac{\Delta n_u}{\bar{r}_c}\frac{\Delta n}{\Delta n_u}} - 1}{e^{\frac{\Delta n_u}{\bar{r}_c}\frac{\Delta n}{\Delta n_u}} + 1}$ is closely approximated by $\frac{1}{2}\frac{\Delta n_u}{\bar{r}_c}\frac{\Delta n}{\Delta n_u}$ if $\Delta n/\bar{r}_c$ is small; therefore,

$$\left(\frac{V_o}{V_u}\right)_{\frac{1}{\bar{r}_c}=0} - \left(\frac{V_o}{V_u}\right) \approx \frac{1}{2}\frac{\Delta n_u}{\bar{r}_c}\frac{\Delta n}{\Delta n_u}\left(\frac{V_o}{V_u}\right)_{\frac{1}{\bar{r}_c}=0} \tag{C2}$$

Use of equation (C2) permits determination of the change in V_o/V_u due to curvature with sufficient accuracy by simple multiplication.

For incompressible flow

$$\frac{\Delta n}{\Delta n_u}\left(\frac{V_o}{V_u}\right)_{\frac{1}{\bar{r}_c}=0} = 1$$

and

$$\left(\frac{V_o}{V_u}\right)_{\frac{1}{\bar{r}_c}=0} - \frac{V_o}{V_u} \approx \frac{1}{2}\frac{\Delta n_u}{\bar{r}_c}$$

Lines of constant values of $\Delta n_u/\bar{r}_c$ for incompressible flow should then be a constant distance from the no-curvature line regardless of the value of $\Delta n/\Delta n_u$.

Since this treatment is for subsonic flow neither V_o nor V may exceed the velocity of sound. The dashed limit lines in figures 3(c) and 4(c) for a value of M_u of 0.85 represent this limitation.

In the derivation (appendix B), n and r_c would be expressed in feet to be consistent with the basic equations. In the derived equation (1) and in the continuity equation (2) it will be noted that Δn appears only as a ratio to another Δn value or as a ratio to r_c. In this case it is only necessary that Δn and r_c be expressed in the same units of length.

APPENDIX D

DETERMINATION OF AN AVERAGE RADIUS OF CURVATURE

A radometer is one means of determining a value of $1/r_c$ of a curve for which this value is not otherwise known. The radometer of figure 5 is similar to the one described in reference 9. It gives, however, a direct reading of $1/r_c$. In figure 5(b), AC is a curve for which the value of $1/r_c$ is to be determined at point B. Points D and E equidistant from B are assumed to be close enough to B for this curve to exclude excessive change in $1/r_c$ between them.

In the nomenclature of the figure, $\frac{c}{b} = \frac{d}{r_c}$ and $\frac{1}{r_c} = \frac{c}{bd}$. Since bd can be a constant for a given radometer, the arc with radius b and center at B can be labeled to read values of $1/r_c$.

In use, the pivot point B is placed on the curve at the point at which $1/r_c$ is to be estimated, the point D of the stationary part is put on the curve, and the movable part is moved until the point E also is on the curve. The value of $1/r_c$ can now be read.

For the instrument used for the example of this report, b was 10 inches and d was 3 inches; $1/r_c$ is expressed in $(\text{inches})^{-1}$ for this radometer.

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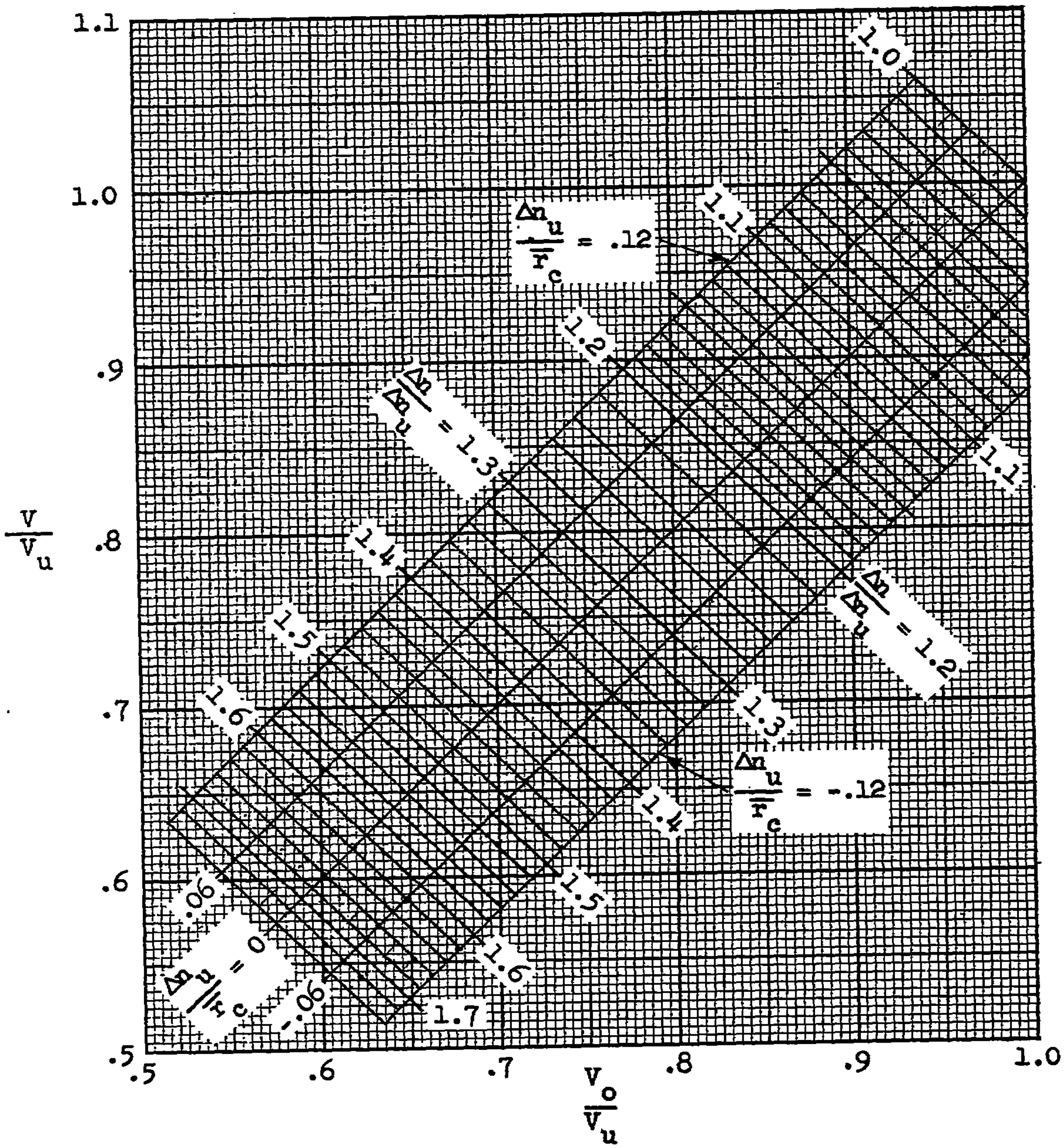
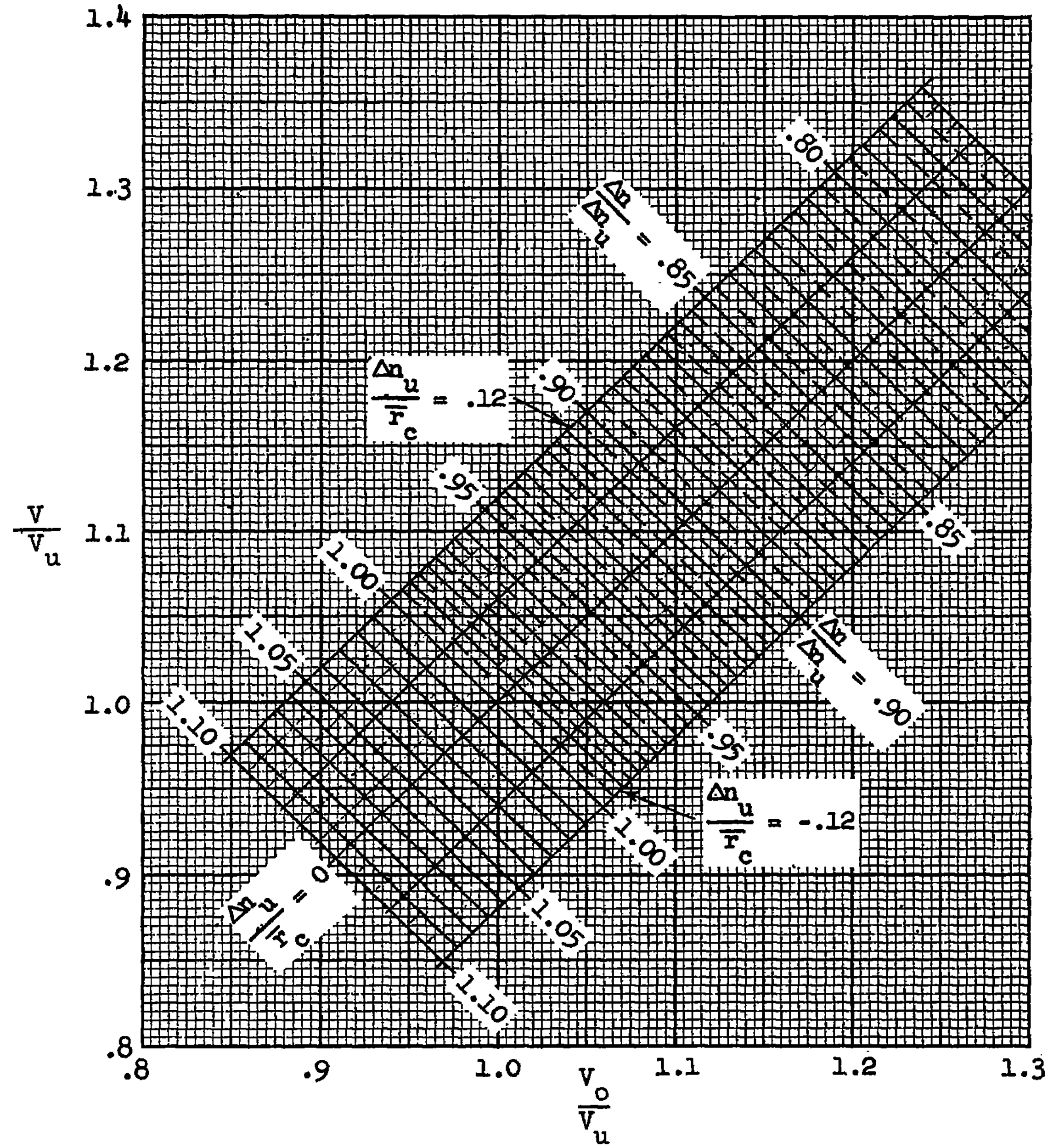


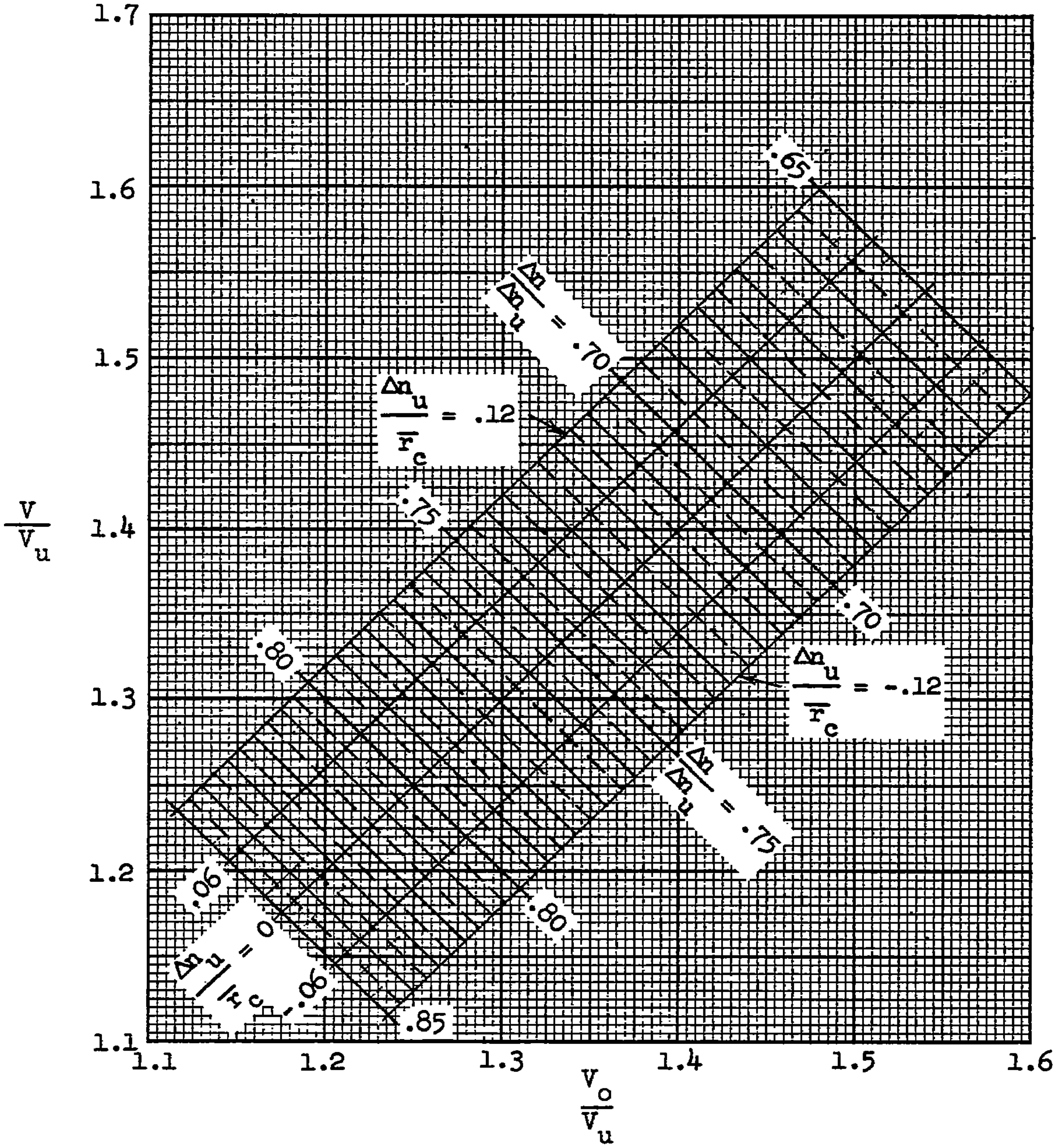
(a) $0.50 < \frac{V_o}{V_u} < 1.00$.

Figure 1.- Design chart. Incompressible flow.



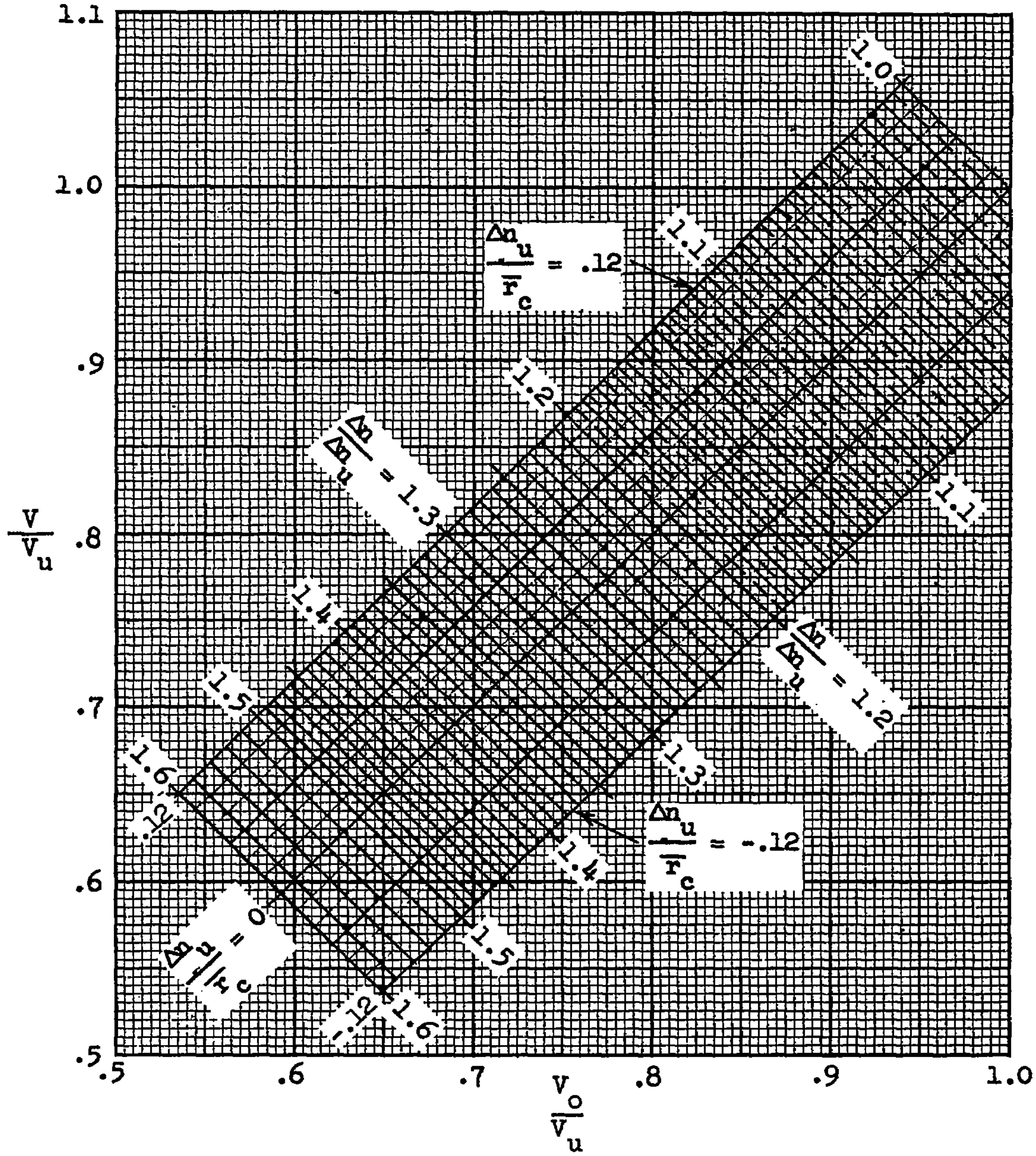
(b) $0.80 < \frac{V_o}{V_u} < 1.30$.

Figure 1.- Continued.



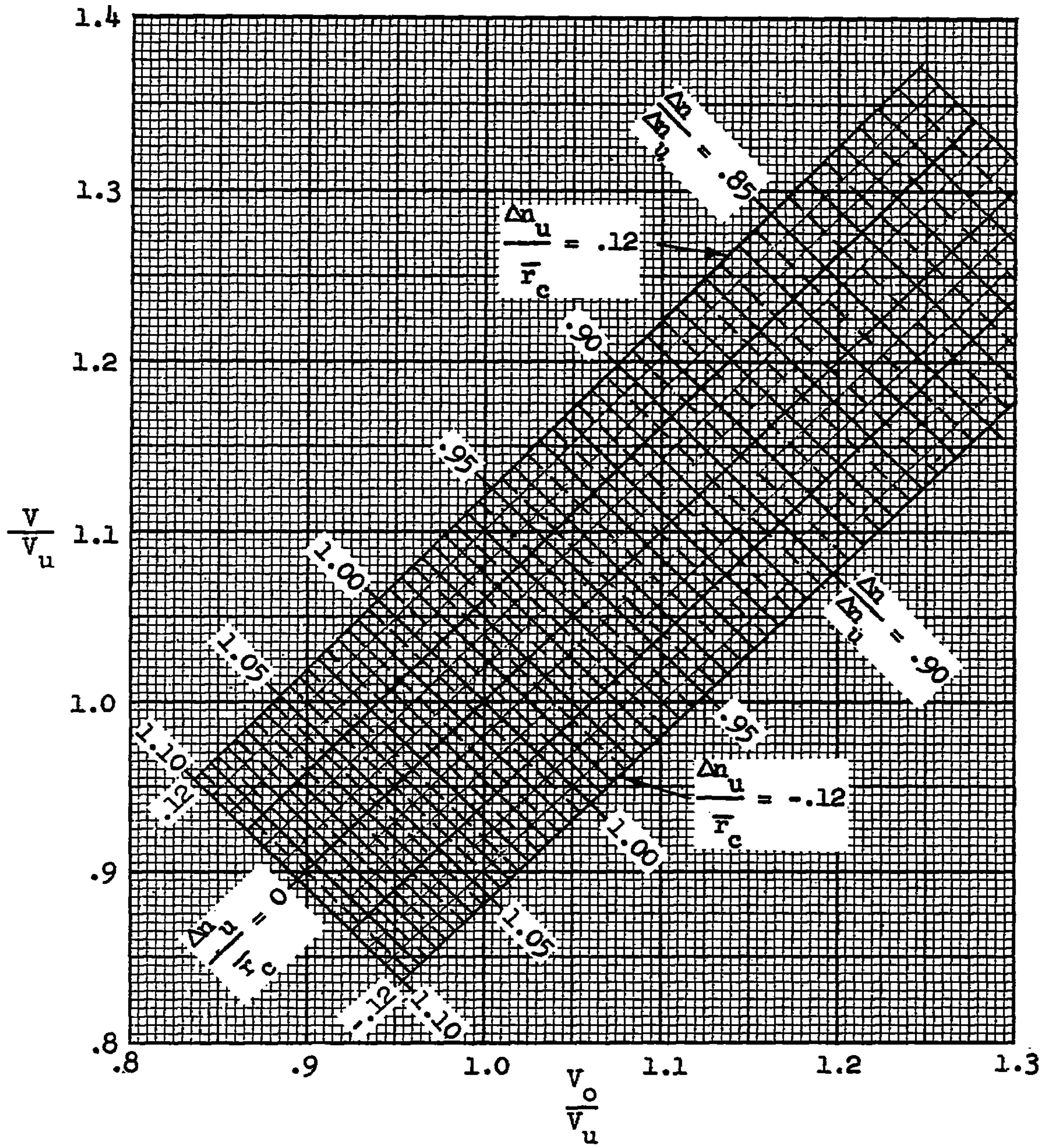
(c) $1.10 < \frac{V_o}{V_u} < 1.60$.

Figure 1.- Concluded.



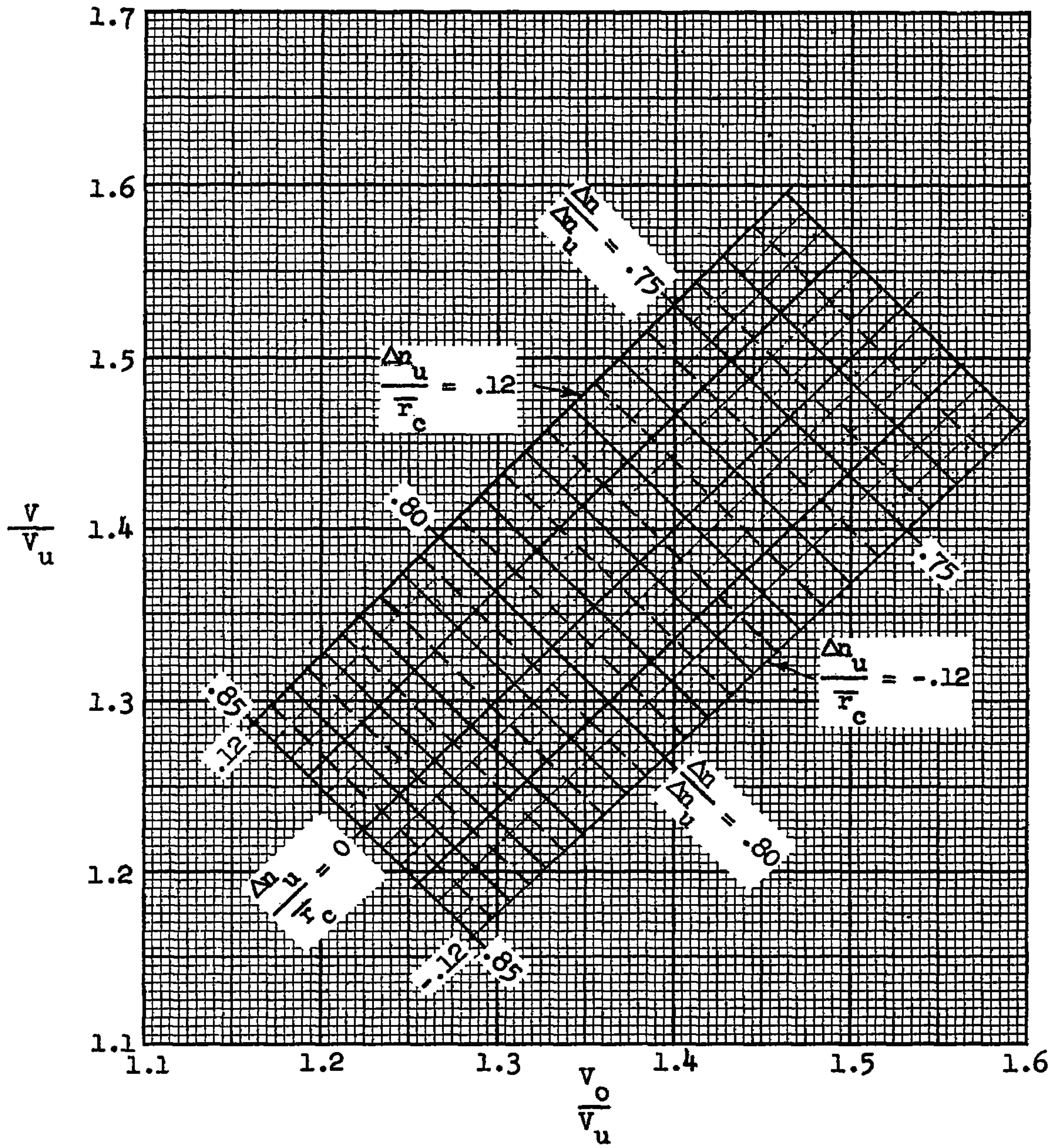
(a) $0.50 < \frac{V_o}{V_u} < 1.00$.

Figure 2.- Design chart. Upstream Mach number, 0.40.



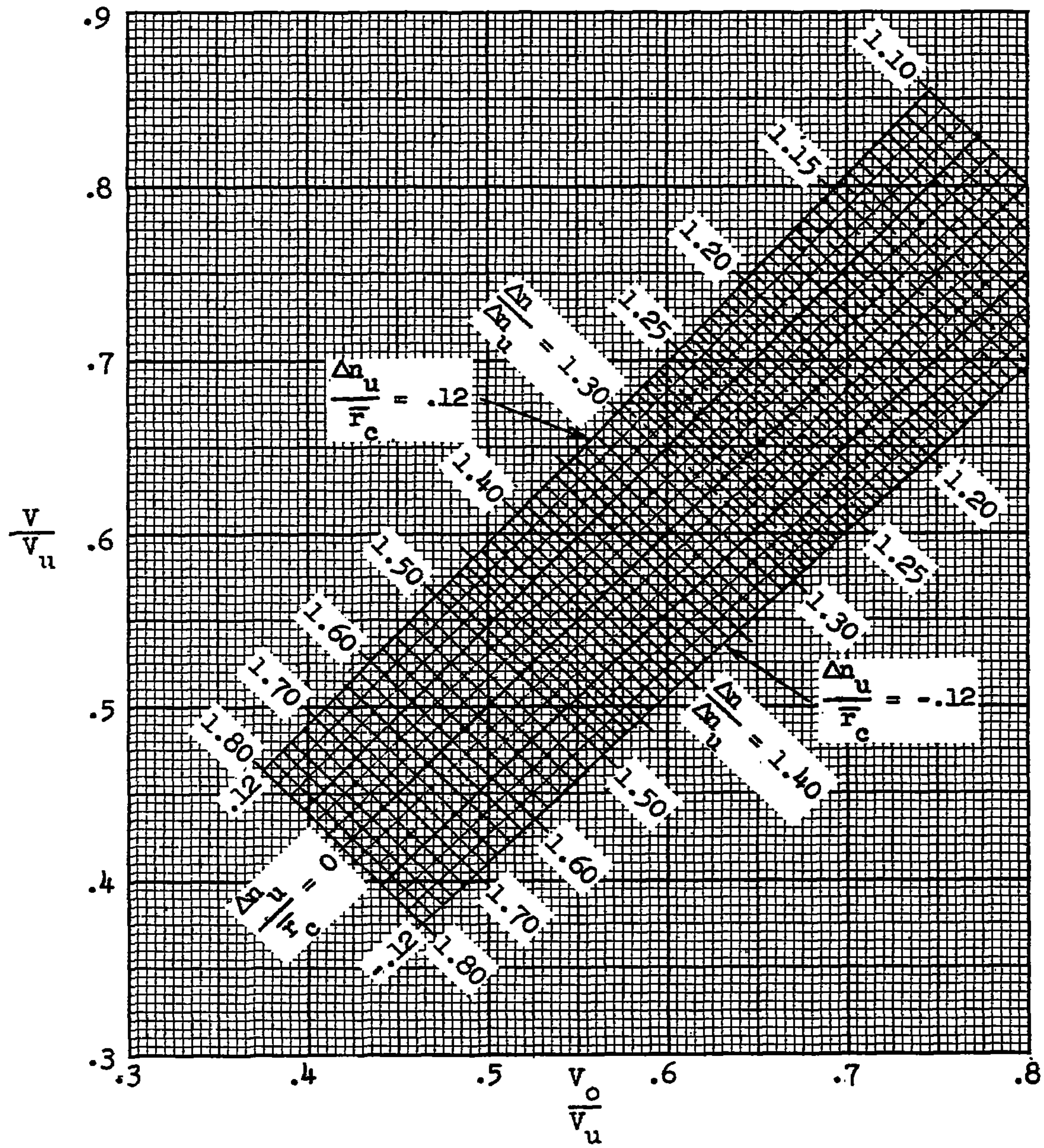
(b) $0.80 < \frac{V_o}{V_u} < 1.30$.

Figure 2.- Continued.



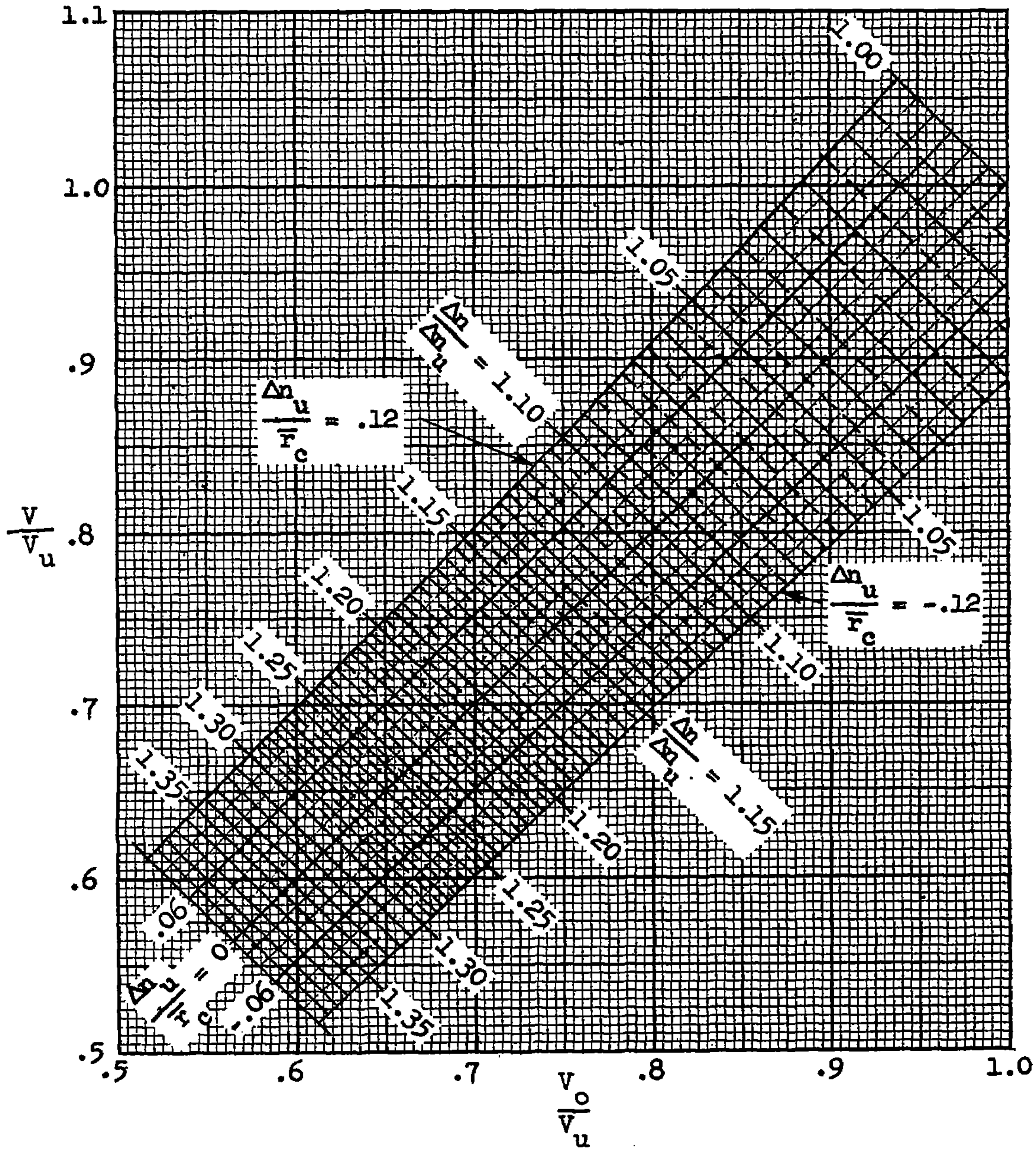
(c) $1.10 < \frac{V_o}{V_u} < 1.60$.

Figure 2.- Concluded.



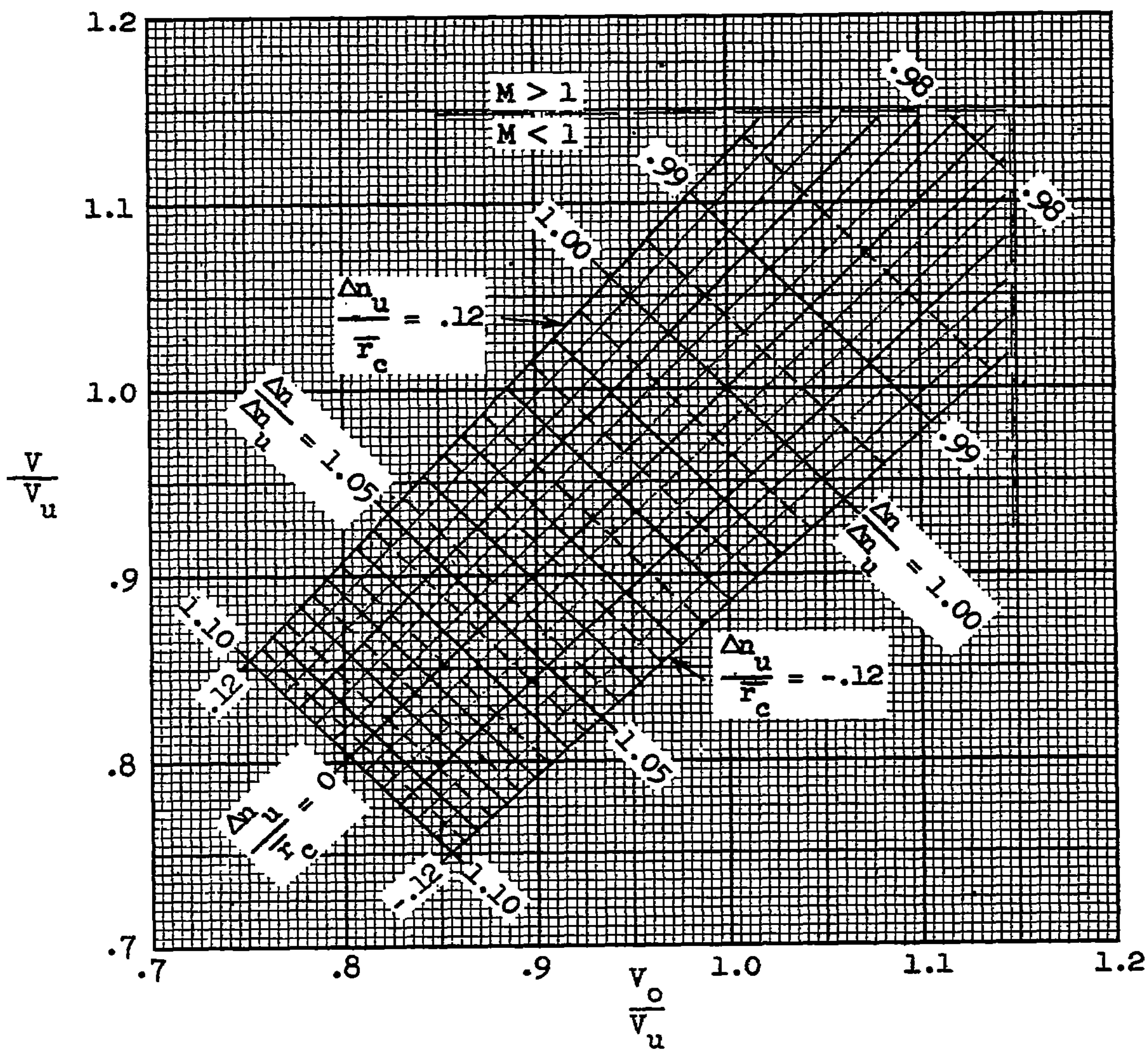
(a) $0.30 < \frac{V_o}{V_u} < 0.80$.

Figure 3.- Design chart. Upstream Mach number, 0.85.



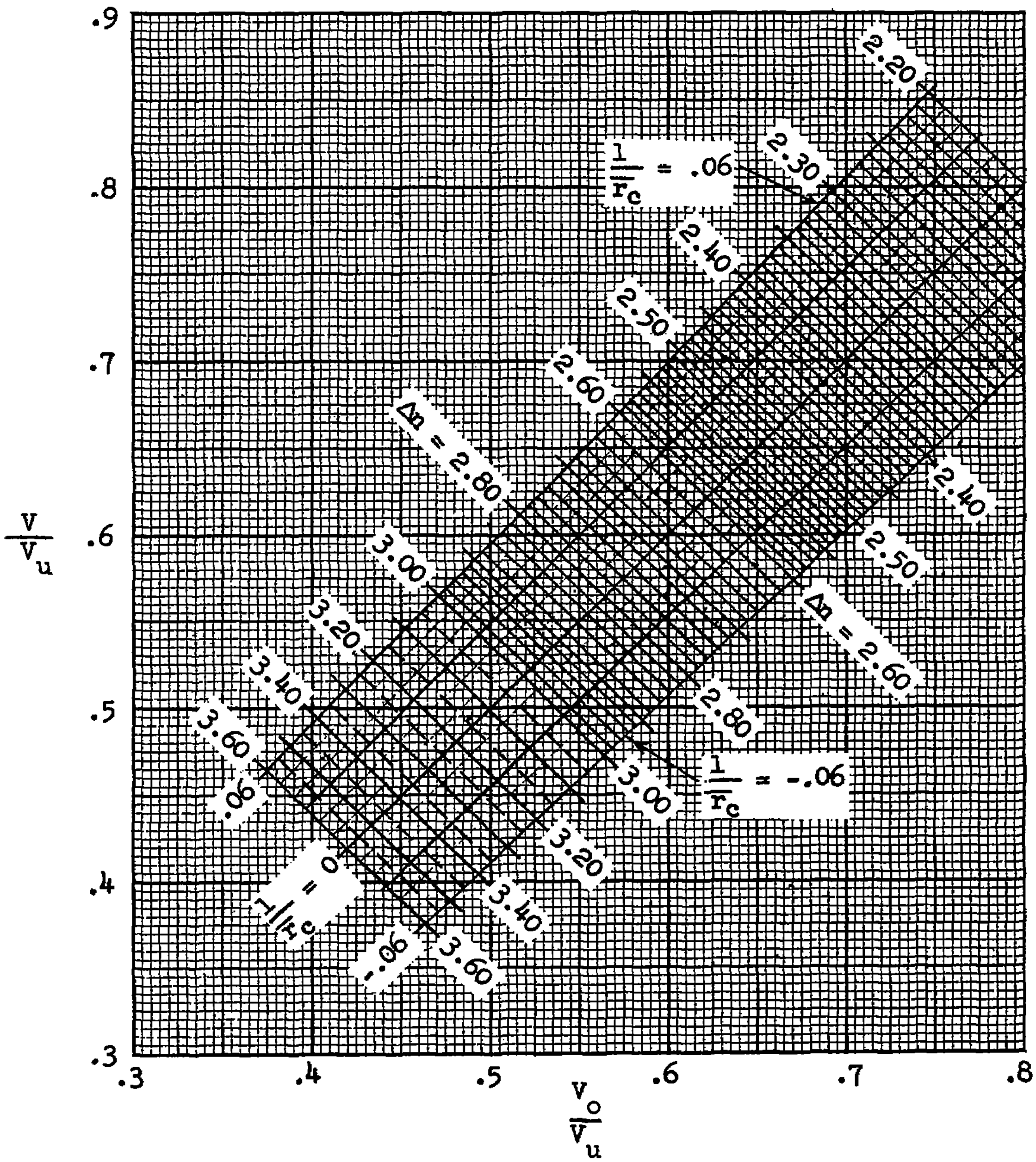
(b) $0.50 < \frac{V_o}{V_u} < 1.00.$

Figure 3.- Continued.



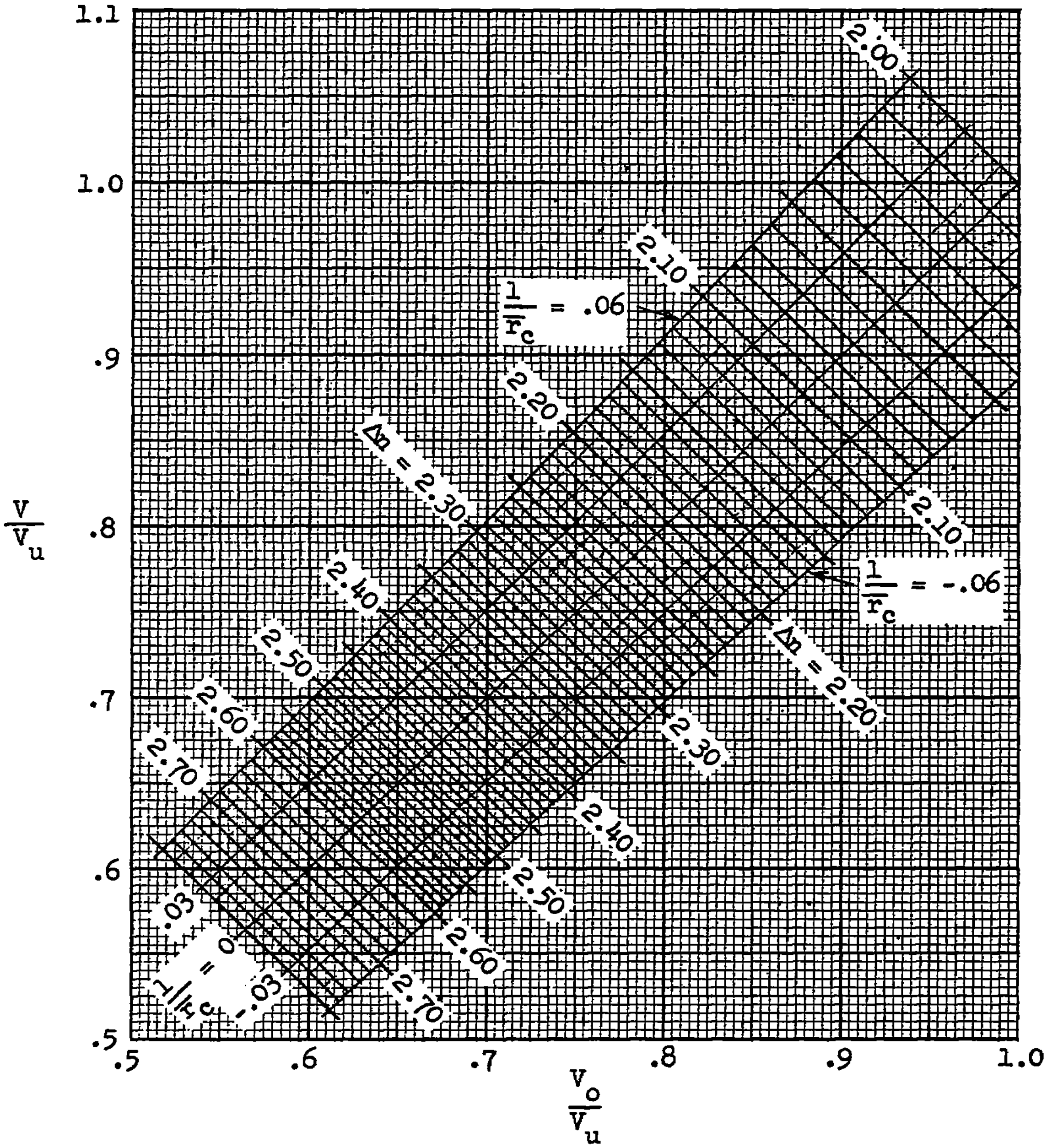
(c) $0.70 < \frac{V_o}{V_u} < 1.20$.

Figure 3.- Concluded.



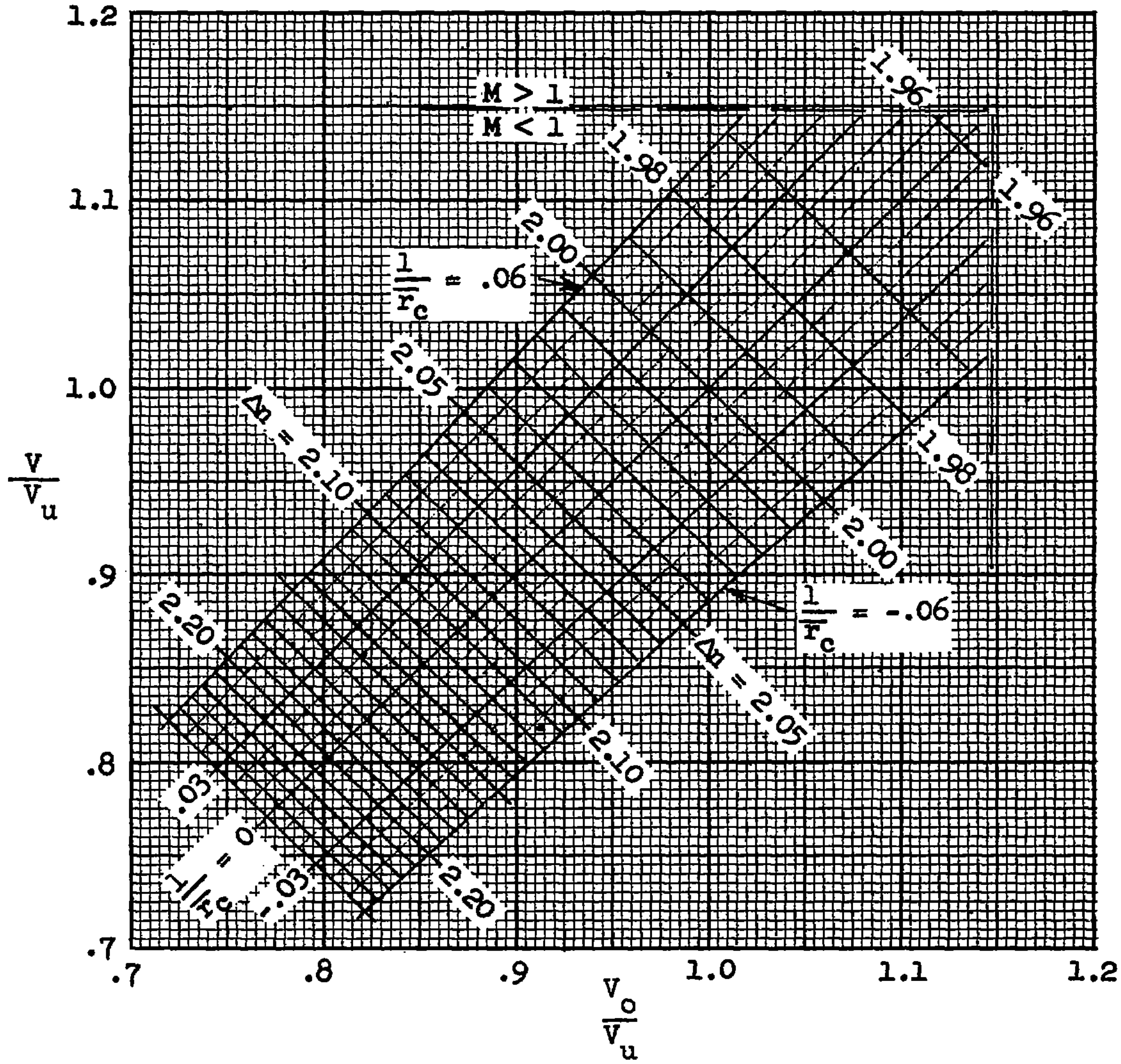
(a) $0.30 < \frac{V_o}{V_u} < 0.80$.

Figure 4.- Specific design chart for upstream width Δn_u of 2.00 inches. Upstream Mach number, 0.85.



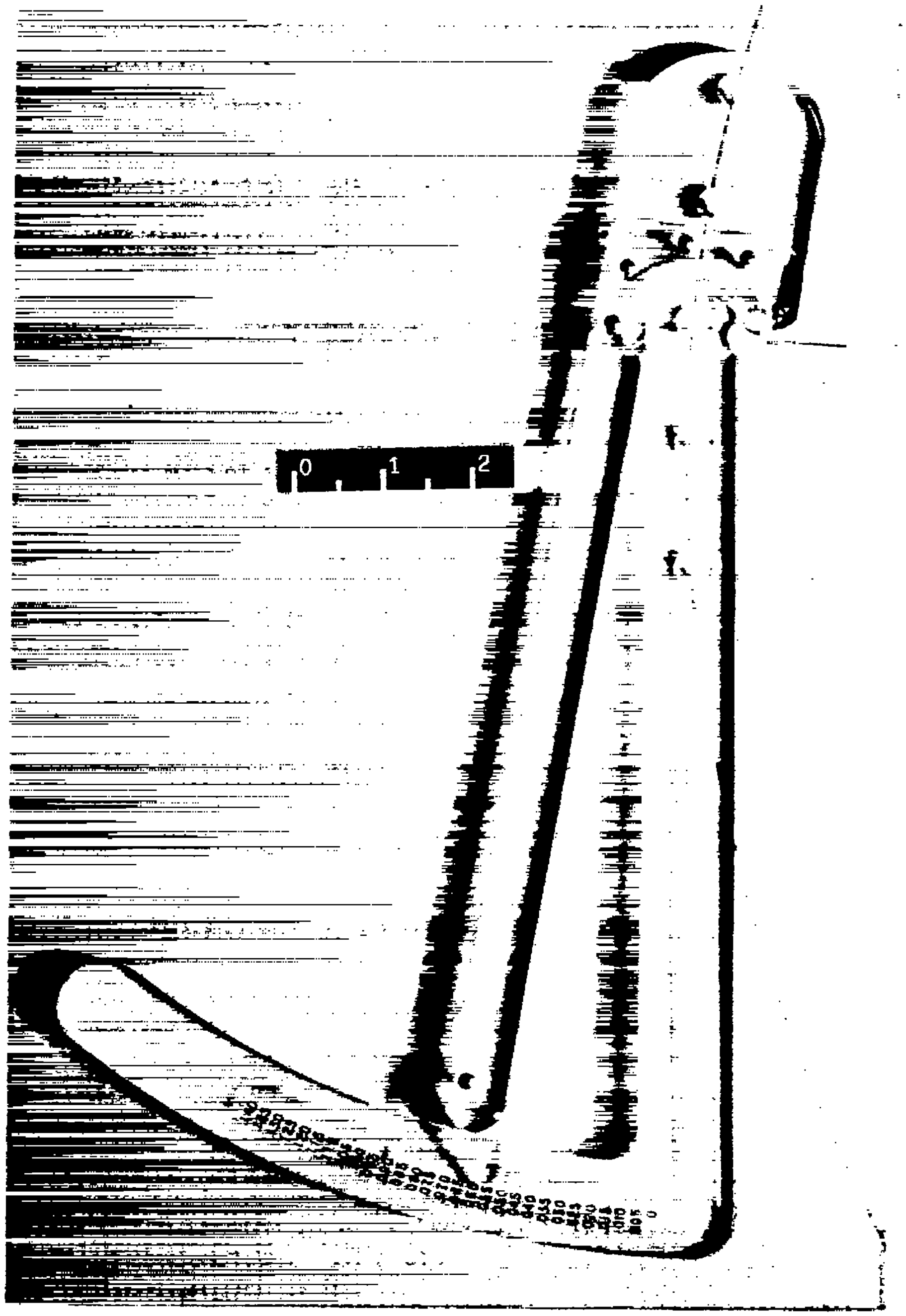
(b) $0.50 < \frac{V_o}{V_u} < 1.00$.

Figure 4.- Continued.



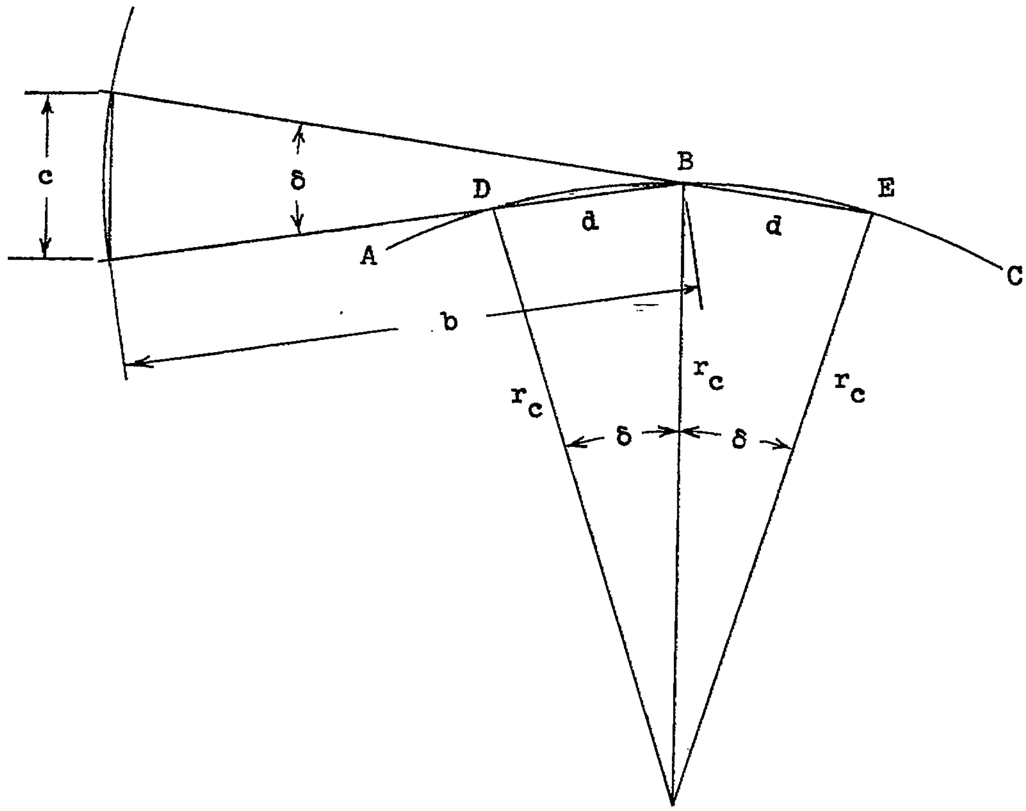
(c) $0.70 < \frac{V_o}{V_u} < 1.20$.

Figure 4.- Concluded.



(a) Photograph of instrument. L-57-2564

Figure 5.- Radometer.



(b) Schematic diagram.

Figure 5.- Concluded.

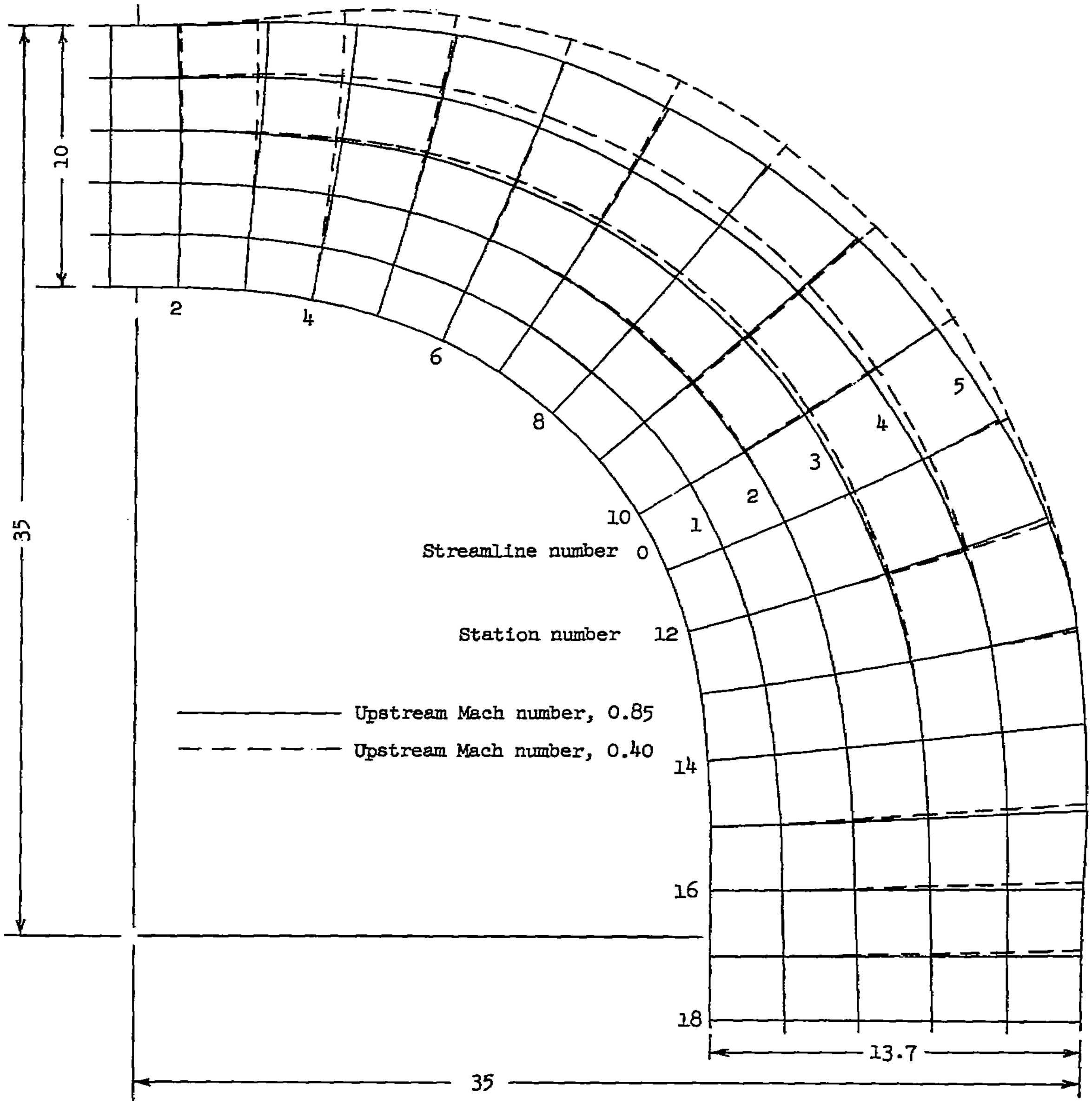
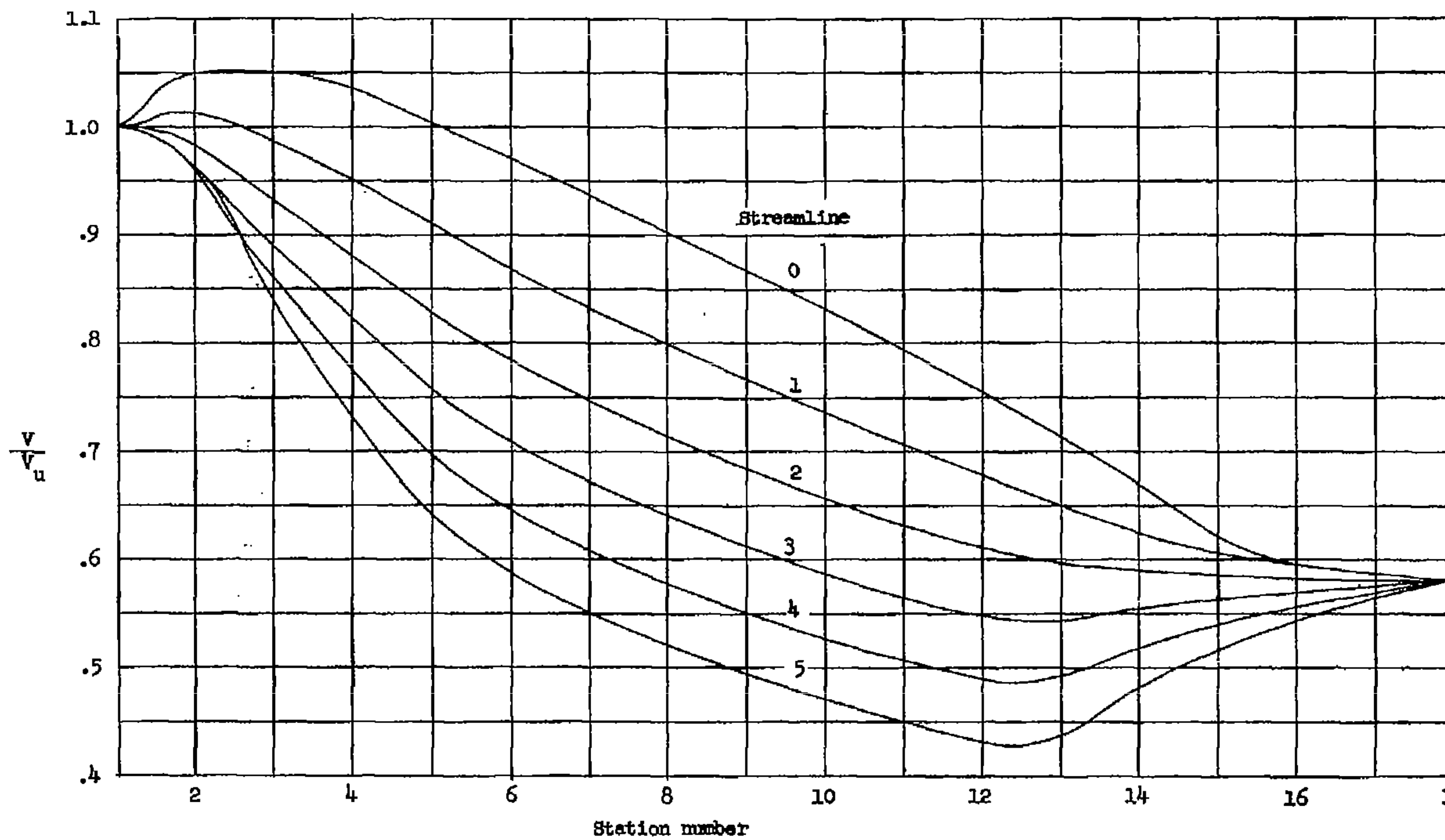
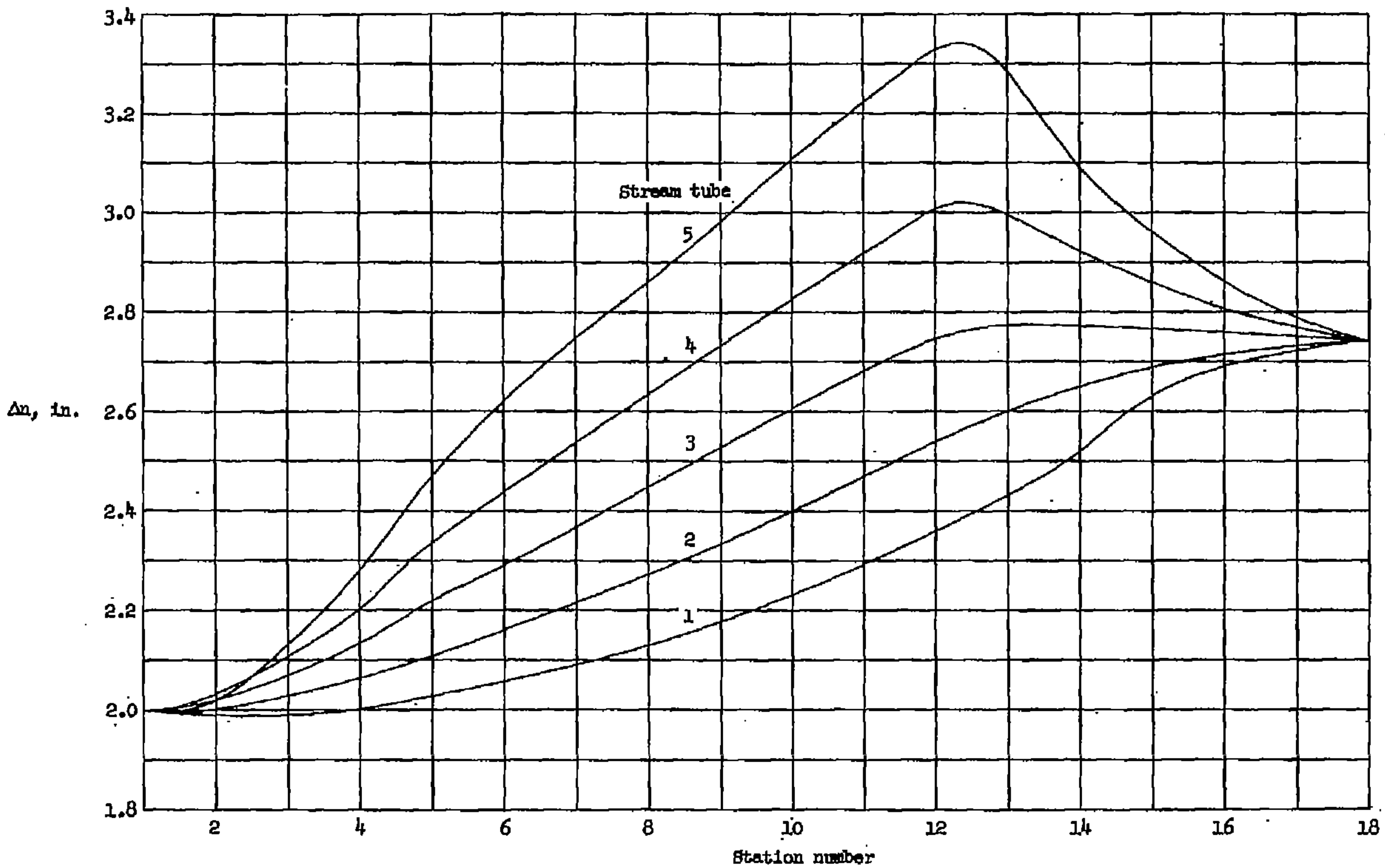


Figure 6.- Elbow contours and streamlines resulting from an arbitrarily assigned inner boundary and an outlet area 1.37 times inlet area.



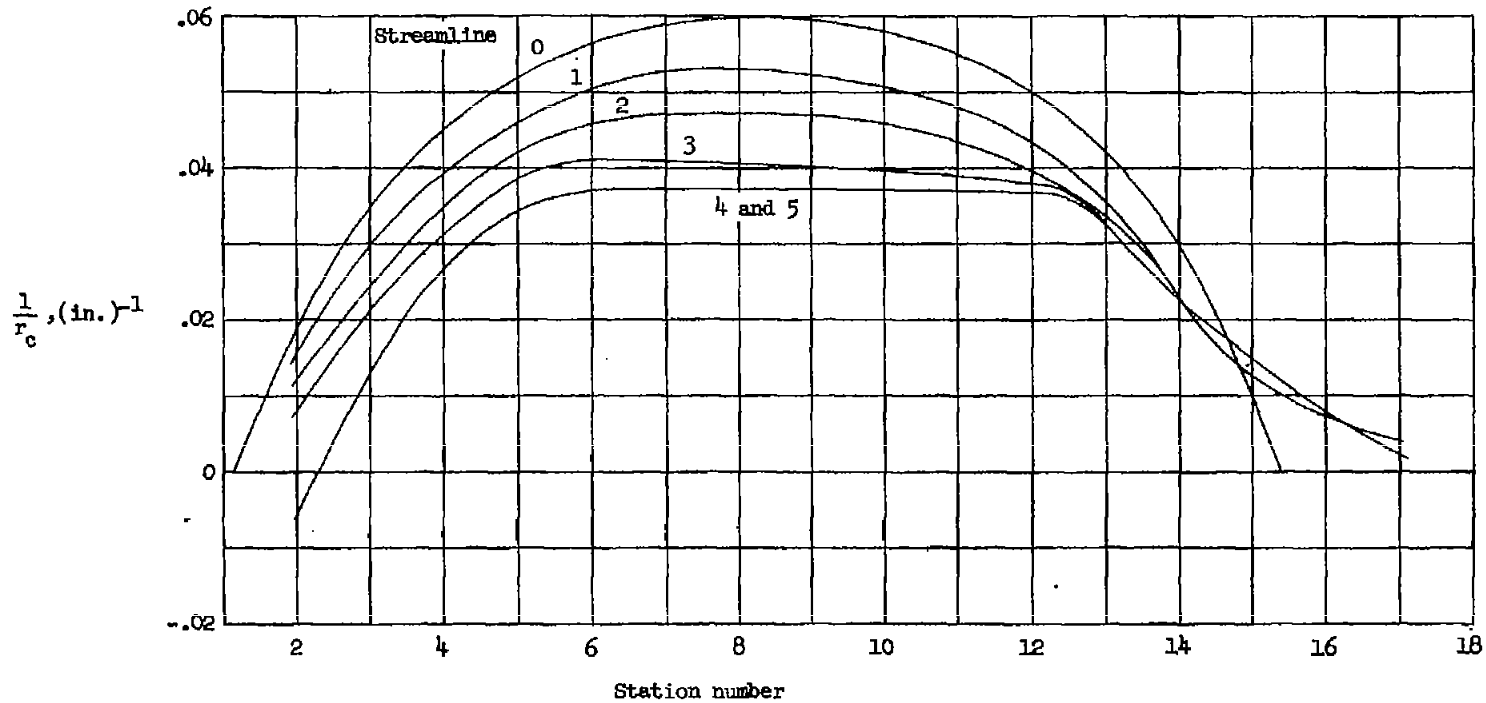
(a) Variation of $\frac{V}{V_u}$.

Figure 7.- Variation of velocity and geometric quantities in a 90° diffusing elbow with an upstream Mach number of 0.85.



(b) Variation of Δn.

Figure 7.- Continued.



(c) Variation of $\frac{1}{r_c}$.

Figure 7.- Concluded.

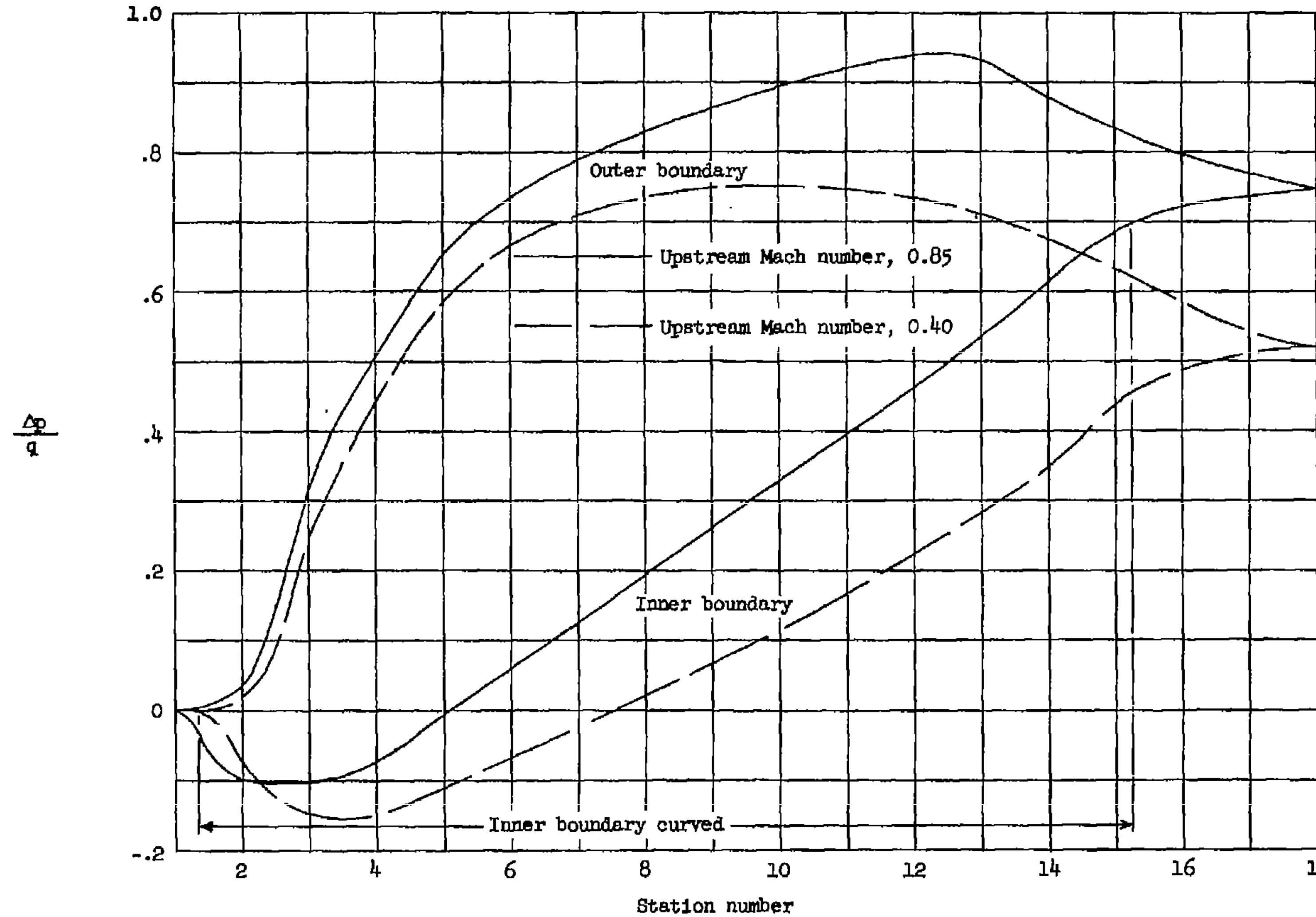


Figure 8.- Comparison of pressure distribution for an upstream Mach number of 0.40 with that for an upstream Mach number of 0.85.

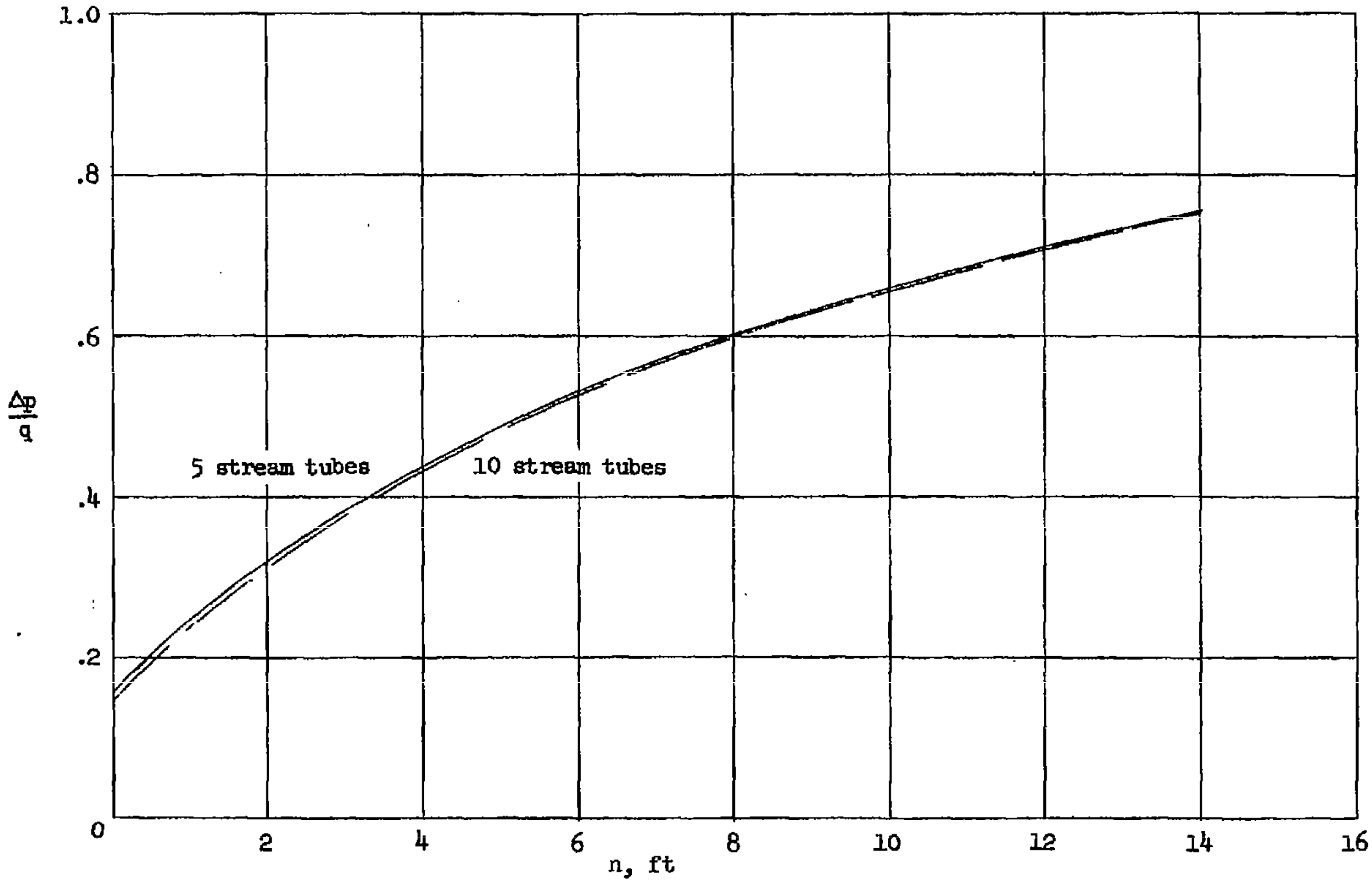


Figure 9.- Comparison of pressure distribution determined for 10 stream tubes with that determined for 5 stream tubes. Upstream Mach number, 0.40.